KOTHARI INTERNATIONAL SCHOOL, NOIDA
PRE BOARD EXAMINATION - 1, SESSION: 2023-24
GRADE: 12 SUBJECT: MATHEMATICS (041)
SET B

DATE& DAY: TUESDAY NOVEMBER 28, 2023
MAXIMUM MARKS: 80 **TIME ALLOTTED: 3 HOUR**
NAME: _____ **ROLL NO: _____**

GENERAL INSTRUCTIONS:

- ***i).*** *This is objective &Subjective Question Paper containing 38 questions.*
- ***ii).*** *This paper contains 20 questions of 1 marks each, 5 questions of 2 marks each and 6 questions of 3 marks each 4 questions of 5 marks each and 3 case/source based questions of 4 marks each.*
- ***iii).*** *1 marks questions are MCQs*
- ***iv).*** *2 and 3 marks questions are Short Answer Type Questions and are to be answered in 50-80 words.*
- ***v).*** *5 marks questions are Long Answer Type Questions and are to be answered in 80-120 words.*
- ***vi).*** *This question paper contains Case/Source Based Questions.*

SECTION – A

Q1. The value of x, for which the matrix $[5 - x\ x + 1\ 2\ 4\ ]$ is singular is **(1)**

a) 1
b) 2
c) 3
d) 4

Q2. If $f(x) = \{\frac{kx}{|x|}\ , if\ x < 0\ 3, if\ x \geq 0$ is continuous at $x = 0$, then the value of k is **(1)**

a) – 3
b) 0
c) 3
d) All real numbers

Q3. The integrating factor of $\frac{dx}{dy} + xcoty = cosy$ is **(1)**

a) $sin\ sin\ x$
b) $siny$
c) $logsiny$
d) e^{siny}

Q4. If the area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq units. Then the value of k will be **(1)**

a) 9
b) 3
c) – 9
d) 6

Q5. The value of $\int_0^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{6-x}} dx$ is **(1)**

a) 3/2

b) 3
c) 2
d) 0

Q6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \left(\frac{dy}{dx}\right)^4$ is **(1)**

a) 2
b) 3
c) 4
d) 1

Q7. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, then the value of k is **(1)**

a) 7/10
b) – 7/10
c) 10/7
d) – 10/7

Q8. If $A = [1\ 0\ -1\ 7\ ]$, then the value of k, if $A^2 = 8A + kI$, is **(1)**

a) – 7
b) 7
c) 5
d) – 5

Q9. If the direction cosines of the line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then **(1)**

a) $0 < c < 1$
b) $c > 2$
c) $c = \pm\sqrt{2}$
d) $c = \pm\sqrt{3}$

Q10. If $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + p\hat{k}$ are perpendicular to each other then the value of p is **(1)**

a) 1
b) – 1
c) 2
d) – 2

Q11. The general solution of the differential equation $e^x dy + (ye^x + 2x)dx = 0$ is **(1)**

a) $xe^y + x^2 = C$
b) $xe^y + y^2 = C$
c) $ye^x + x^2 = C$
d) $ye^y + x^2 = C$

Q12. If A and B are invertible square matrices of the same order, then which of the following is not correct? **(1)**

a) $\left|AB^{-1}\right| = \frac{|A|}{|B|}$
b) $\left|(AB)^{-1}\right| = \frac{1}{|A||B|}$
c) $(AB)^{-1} = B^{-1}A^{-1}$
d) $(A+B)^{-1} = A^{-1} + B^{-1}$

Q13. The vector in the direction of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has a magnitude of 8 units is **(1)**

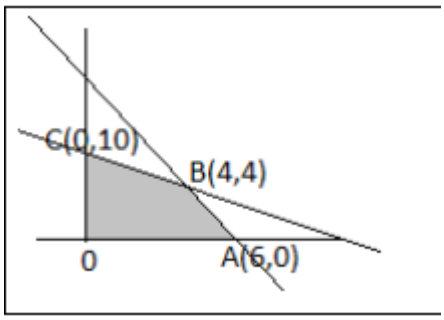
a) $8\vec{a}$
b) $24\vec{a}$
c) $\frac{8}{3}\vec{a}$

d) $\frac{3}{8}\vec{a}$

Q14. If A is a square matrix of order 3×3, then $|kA|$ is equal to **(1)**

a) $k|A|$
b) $k^2|A|$
c) $k^3|A|$
d) $k^4|A|$

Q15. The corner points of the shaded bounded feasible region of an LPP are (0, 10), (4,4) and (6, 0) as shown in the figure. The maximum value of the objective function Z = 5x + 3y is **(1)**

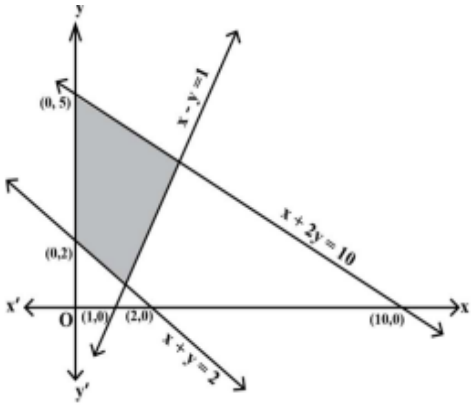


a) 30
b) 32
c) 28
d) 34

Q16. If $A = [a_{ij}]_{m\times n}$, then A' is equal to **(1)**

a) $[a_{ij}]_{m\times n}$
b) $[a_{ij}]_{n\times m}$
c) $[a_{ji}]_{m\times n}$
d) $[a_{ji}]_{n\times m}$

Q17. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below **(1)**



Which of the following is not a constraint to the given Linear Programming Problem?

a) $x + y \geq 2$
b) $x + 2y \leq 10$
c) $x - y \geq 1$
d) $x - y \leq 1$

Q18. If $A = [a\ 0\ 0\ 0\ a\ 0\ 0\ 0\ a\ ]$, then the value of $|adj.A|$ is **(1)**

a) a^{27}
b) a^6
c) a^9
d) a^2

Assertion – Reason based questions

In questions 7 and 8, a statement of assertion (A) is followed by a statement of Reason (R) is given. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion**: The domain of $cot^{-1}x$ is the set of all real numbers. **(1)**

Reason: $cot^{-1}(-1) = -\pi/4$

Q20. **Assertion**: The function $f: \{1,2,3,4\} \to \{x,\ y,z,p\}$ defined by $f = \{(1,x),(2,y),(3,z)\}$ is a bijective function. **(1)**

Reason: The function $f: \{1,2,3\} \to \{x,\ y,z,p\}$ such that $f = \{(1,x),(2,y),(3,z)\}$ is a one – one function.

SECTION B

Q21. Evaluate: $\int_{-1}^{1} \ log\ log\ \left(\frac{2-x}{2+x}\right)\ dx$ **(2)**

Q22. Find the value of α when projection of $\vec{a} = \alpha\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units. **(2)**

Q23. Find the principal value of $sin^{-1}\left(sin\frac{3\pi}{5}\right)$ **(2)**

Q24. The side of an equilateral triangle is increasing at the rate of 2cm/sec. At what rate area will increase when the side is 20 cm. **(2)**

Q25. Find the intervals in which the function f given by $f(x) = x^3 - 6x^2 + 9x + 15$ is strictly increasing. **(2)**

SECTION C

Q26. Solve the following Linear Programming Problem graphically: **(3)**

Minimize: $z = x + 2y$,

subject to the constraints: $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0.$

Q27. If $x = a\ sec^3\theta, y = a\ tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **(3)**

Q28. Find the equation of the line passing through the point $P(1, 2, -4)$ and perpendicular to th two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ **(3)**

Q29. Evaluate: $\int \frac{x}{(x-1)^2(x+2)} dx$ **(3)**

Q30. Evaluate: $\int_0^3 (|x-1| + |x-2|) dx$ **(3)**

Q31. Solve the differential equation $cos^2 x \frac{dy}{dx} + y = tan\ tan\ x$ **(3)**

SECTION D

Q32. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ **(5)**

Q33. Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ **(5)**

Q34. Define the relation R in the set $N \times N$ as follows: For (a, b), (c, d) $\in N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$. **(5)**

Q35. Solve the system of equations by matrix method. **(5)**

$x - 2y - 2z = 9 \qquad x - y + z = 4 \qquad 2x + y + 3z = 1$

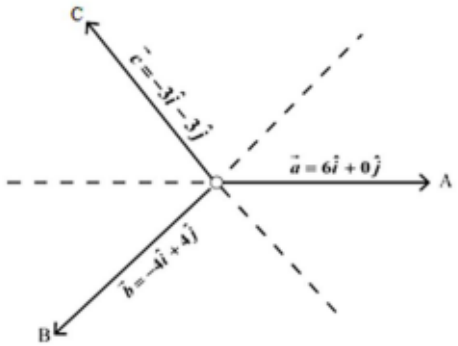
SECTION E

Q36. Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ kN$,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ kN$,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ kN$,



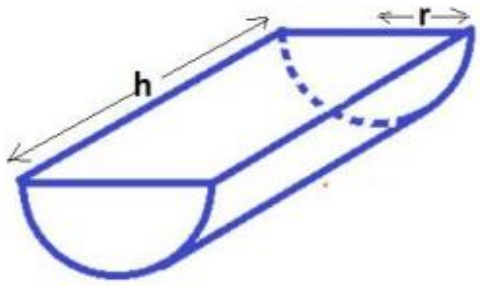
(i) What is the magnitude of the force of Team A? **(1)**

(ii) Which team will win the game? **(1)**

(iii) Find the magnitude of the resultant force exerted by the teams. **(2)**

Q37. An engineer wants to cast metal into solid half cylinders of given volume V with a rectangular base and semi-circular ends.

Based on the information provided answer the following.



i. Find the total surface area 'S' of the casted half cylinder. **(1)**

ii. Find the expression for total surface area S in terms of V, volume of half cylinder and r. **(1)**

iii. Find the minimum surface area of the half cylinder. **(2)**

Q38. The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - x^2/2$ where x is the number of days exposed to sunlight.

(i)What is the number of days it will take for the plant to grow to the maximum height? **(2)**

(ii)What is the maximum height of the plant? **(2)**