KOTHARI INTERNATIONAL SCHOOL, NOIDA
PRE BOARD EXAMINATION - 1, SESSION: 2023-24
GRADE: 12 SUBJECT: MATHEMATICS (041)
SET A

DATE& DAY: TUESDAY NOVEMBER 28, 2023
MAXIMUM MARKS: 80 **TIME ALLOTTED: 3 HOUR**
NAME: _____ **ROLL NO: _____**

GENERAL INSTRUCTIONS:

- ***i).*** *This is objective &Subjective Question Paper containing 38 questions.*
- ***ii).*** *This paper contains 20 questions of 1 marks each, 5 questions of 2 marks each and 6 questions of 3 marks each 4 questions of 5 marks each and 3 case/source based questions of 4 marks each.*
- ***iii).*** *1 marks questions are MCQs*
- ***iv).*** *2 and 3 marks questions are Short Answer Type Questions and are to be answered in 50-80 words.*
- ***v).*** *5 marks questions are Long Answer Type Questions and are to be answered in 80-120 words.*
- ***vi).*** *This question paper contains Case/Source Based Questions.*

SECTION – A

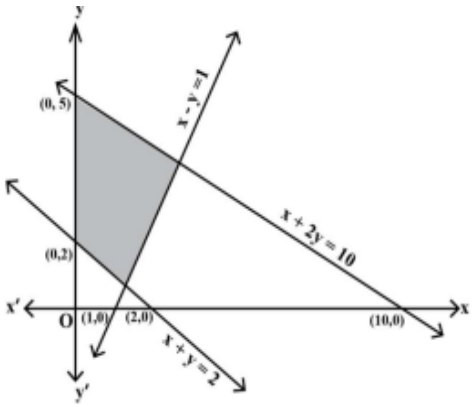
Q1. If $A = [a\ 0\ 0\ 0\ a\ 0\ 0\ 0\ a\ ]$, then the value of $|adj.A|$ is **(1)**

a) a^{27}
b) a^6
c) a^9
d) a^2

Q2. If $|x+1\ x-1\ x-3\ x+2\ | = |4\ -1\ 1\ 3\ |$, then the value of x is **(1)**

a) 2
b) 3
c) 0
d) – 4

Q3. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below **(1)**



Which of the following is not a constraint to the given Linear Programming Problem?

a) $x + y \geq 2$
b) $x + 2y \leq 10$

c) $x - y \geq 1$
d) $x - y \leq 1$

Q4. If $f(x) = \{\frac{kx}{|x|}, if\ x < 0\ 3, if\ x \geq 0$ is continuous at $x = 0$, then the value of k is **(1)**

a) – 3
b) 0
c) 3
d) All real numbers

Q5. The integrating factor of $\frac{dx}{dy} + xcoty = cosy$ is **(1)**

a) $sin\ sin\ x$
b) $siny$
c) $logsiny$
d) e^{siny}

Q6. If $A = [a_{ij}]_{m\times n}$, then A' is equal to **(1)**

a) $[a_{ij}]_{m\times n}$
b) $[a_{ij}]_{n\times m}$
c) $[a_{ji}]_{m\times n}$
d) $[a_{ji}]_{n\times m}$

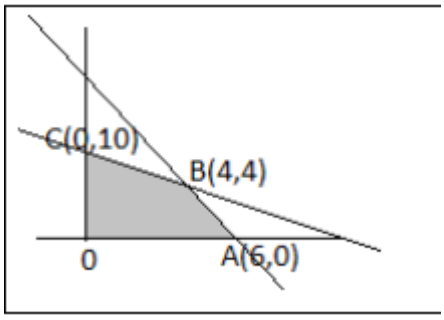
Q7. If the area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq units. Then the value of k will be **(1)**

a) 9
b) 3
c) – 9
d) 6

Q8. If A and B are invertible square matrices of the same order, then which of the following is not correct? **(1)**

a) $\left|AB^{-1}\right| = \frac{|A|}{|B|}$
b) $\left|(AB)^{-1}\right| = \frac{1}{|A||B|}$
c) $(AB)^{-1} = B^{-1}A^{-1}$
d) $(A + B)^{-1} = A^{-1} + B^{-1}$

Q9. The corner points of the shaded bounded feasible region of an LPP are (0, 10), (4,4) and (6, 0) as shown in the figure. The maximum value of the objective function Z = 5x + 3y is **(1)**



a) 30
b) 32
c) 28

d) 34

Q10. The value of $\int_{1/e}^{e} \frac{dx}{x(loglog\ x)^{1/3}}$ is **(1)**

a) e
b) $\frac{1}{e}$
c) 0
d) 1

Q11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \left(\frac{dy}{dx}\right)^4$ is **(1)**

a) 2
b) 3
c) 4
d) 1

Q12. The vector in the direction of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has a magnitude of 8 units is **(1)**

a) $8\vec{a}$
b) $24\vec{a}$
c) $\frac{8}{3}\vec{a}$
d) $\frac{3}{8}\vec{a}$

Q13. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, then the value of k is **(1)**

a) 7/10
b) – 7/10
c) 10/7
d) – 10/7

Q14. If A is a square matrix of order 3×3, then $|kA|$ is equal to **(1)**

a) $k|A|$
b) $k^2|A|$
c) $k^3|A|$
d) $k^4|A|$

Q15. If $A = [1\ 0\ -1\ 7\ ]$, then the value of k, if $A^2 = 8A + kI$, is **(1)**

a) – 7
b) 7
c) 5
d) – 5

Q16. If the direction cosines of the line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then **(1)**

a) $0 < c < 1$
b) $c > 2$
c) $c = \pm\sqrt{2}$
d) $c = \pm\sqrt{3}$

Q17. The general solution of the differential equation $ydx - xdy = 0$ is of the form **(1)**

a) $xy = c$
b) $x = cy^2$
c) $y = cx$
d) $y = cx^2$

Q18. If $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + p\hat{k}$ are perpendicular to each other then the value of p is **(1)**

a) 1
b) – 1
c) 2
d) – 2

Assertion – Reason based questions

In questions 7 and 8, a statement of assertion (A) is followed by a statement of Reason (R) is given. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion**: The function $f:\{1,2,3,4\} \rightarrow \{x,\ y,z,p\}$ defined by $f = \{(1,x),(2,y),(3,z)\}$ is a bijective function. **(1)**

Reason: The function $f:\{1,2,3\} \rightarrow \{x,\ y,z,p\}$ such that $f = \{(1,x),(2,y),(3,z)\}$ is a one – one function.

Q20. **Assertion**: The domain of $cot^{-1}x$ is the set of all real numbers. **(1)**

Reason: $cot^{-1}(-1) = -\pi/4$

SECTION B

Q21. Find the principal value of $sin^{-1}\left(sin\frac{3\pi}{5}\right)$ **(2)**

Q22. A stone is dropped in to a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when radius of the circular wave is 10cm, how fast is the enclosed area increasing? **(2)**

Q23. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees. **(2)**

Q24. Evaluate: $\int_{-1}^{1} \quad log\ log\ \left(\frac{2-x}{2+x}\right)\ dx$ **(2)**

Q25. Find the value of α when projection of $\vec{a} = \alpha\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units. **(2)**

SECTION C

Q26. Evaluate: $\int \quad \frac{3x-5}{(x-1)^2(x-2)}dx$ **(3)**

Q27. Find the equation of the line passing through the point $P(1,2,-4)$ and perpendicular to th two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ **(3)**

Q28. Evaluate: $\int_0^{\pi/4} \quad log\ log\ (1 + tanx)dx$ **(3)**

Q29. Solve the differential equation $cos^2x\frac{dy}{dx} + y = tan\ tan\ x$ **(3)**

Q30. Solve the following Linear Programming Problem graphically: **(3)**

Minimize: $z = x + 2y$,

subject to the constraints: $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0.$

Q31. If $x = a\ sec^3\theta, y = a\ tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **(3)**

SECTION D

Q32. Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ **(5)**

Q33. Let a relation R defined on A = {1,2,3,4,5,6,7,8,9,10} be defined as R = {(a,b) : |a - b| is an even number }. Prove that the relation is equivalence. Hence find all elements which are related to 3 **(5)**

Q34. Solve the system of equations by matrix method. **(5)**

$x - 2y - 2z = 9 \qquad x - y + z = 4 \qquad 2x + y + 3z = 1$

Q35. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ **(5)**

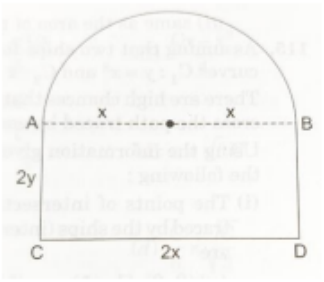
SECTION E

Q36. The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - x^2/2$ where x is the number of days exposed to sunlight.

(i)What is the number of days it will take for the plant to grow to the maximum height? **(2)**

(ii)What is the maximum height of the plant? **(2)**

Q37.



Mr.Shashi who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semicircular opening. This window is having a perimeter of 10m as shown below :Based on the above information answer the following questions.

(i) If 2x and 2y are the length and breadth of the rectangular portion of the window, then find the relation between variables **(1)**

(ii) Find the combined area A of rectangular region and semi-circular region of the window in terms of x. **(1)**

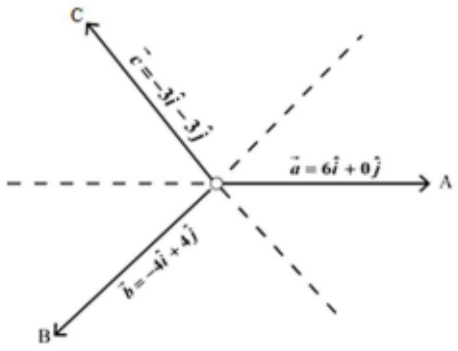
(iii)Find the maximum value of area A, of the whole window. **(2)**

Q38. Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ kN$,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ kN$,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ kN$,



(i) What is the magnitude of the force of Team A ? **(1)**

(ii) Which team will win the game? **(1)**

(iii) Find the magnitude of the resultant force exerted by the teams. **(2)**